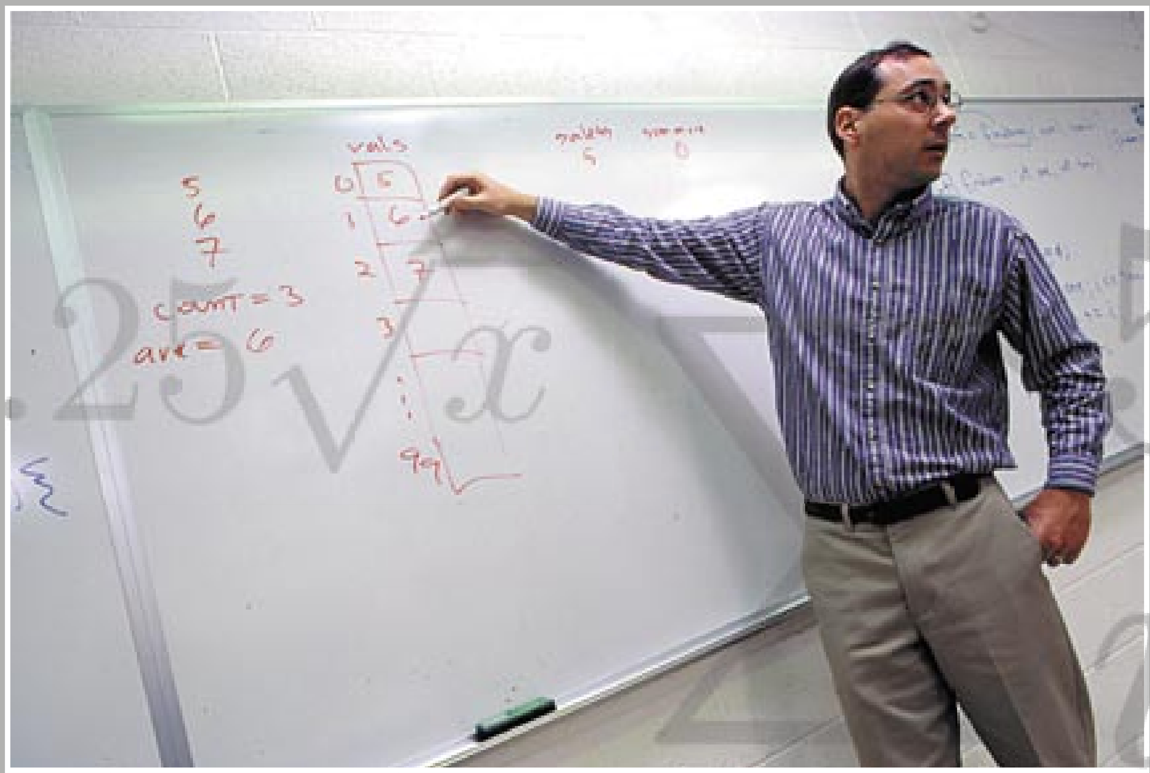
MATH FOR NON-MATHERS

Using math in everyday life

By Michael Gousie



I often hear the phrase, "I'm not a math person," and I have seen many a good student simply cringe at the thought of having to take "Statistics" or another mathematics course. For a math and computer science professor, this is troubling. I started thinking.

There are several factors in explaining this behavior. One problem is that as a nation, we simply do not expect much of our students as far as mathematics is concerned. In many countries, calculus is a requirement at the high school level. In the U.S., algebra is thought of as being difficult and perhaps not even useful. Contrast this to my first grade experience in Switzerland. I still have my math workbook (Rechenbüchlein, in German) in which I had to solve basic algebraic equations such as:

$$4/5 \text{ of } 10 - 2/3 \text{ of } 9 + \_\_\_ = 11$$

This one problem uses fractions and algebra to arrive at the answer of 9.

The point is that while we may not expect a first-grader to be able to handle this—I don't necessarily believe that a first-grader needs to do this—we should expect high school students to be able to handle algebra and geometry, and college-level students to be able to handle mathematics at an appropriate level.

A second problem is that mathematics is fraught with notation. This is necessary so that there is a consistent way of representing ideas. Although notation can be a little daunting, it's really no different than a student having to learn special symbols in a French course or, even better, learning a whole new alphabet when taking Greek. It just takes a little bit of effort, which shouldn't be discouraging to a college student.

Why, even reading English can take some effort:

> Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns—the ones we don't know we don't know. And if one looks throughout the history of our country and other free countries, it is the latter category that tend to be the difficult ones.
>
> —*Donald Rumsfeld*

Yet another issue is the fact that mathematics is inherently sequential. One cannot simply jump to calculus without a sufficient background. A "bump in the road" can have a detrimental effect for years to come. I myself had a terrible Algebra I teacher, and I then struggled through Algebra II as well. But I knew that math was important and was necessary for future work in any science discipline, so I continued to take math courses.

Perhaps the biggest obstacle to having a good attitude toward mathematics is the notion that it's somehow too difficult, very abstract and not at all useful. Admittedly, higher mathematics can become quite involved, but general math—from algebra to statistics to, yes, calculus—is presented to us frequently and we often don't even realize. We just do it. People use math all the time, but either think nothing of it, or don't bother to really understand what's going on, even if it's not that difficult. What follows are a few examples.

Algebra

Algebra is often thought of as a four-letter word, that word usually not being "nice." One major component of algebra is the notion of a function, an example of which is

$$f(x) = x^2$$

Here's where notation makes things scary. All this means is

$$y = x^2$$

In other words, if you plug any value for x into the right side, the result "of the function" is y. If $x = 3$, then $f(x) = y = 9$.

See? This isn't hard. It's just a matter of taking a moment to understand the notation.

Another major idea in algebra is solving for an unknown. An example is

$$y = x + 0.05x + 0.15x$$

Looking at this, you might be thinking it's time to stop reading this ridiculous article. But let me rewrite the above this way:

	Food	50.00
	Tax@5%	2.50
+	Tip@15%	7.50
	Total	60.00

Here x represents food, $0.05x$ represents the tax, which is 5 percent of the food cost ($2.50), and $0.15x$ is the tip, or 15 percent of the food cost ($7.50).

Add them all up and you get y, which is your total bill. All of us do this calculation frequently.

The hardest part of the above is figuring out the darn tip amount. We know it's 15 percent (or 20 percent if you're not as cheap as I am) of the food cost. Algebraically, it's *tip* = 0.15 x *food*. Here's the beauty of math: If you really understand what's going on, you can see shortcuts. For example, since the state tax is 5 percent, you can simply multiply the tax on the bill by 3 to get 15%, or the tip amount (or multiply by 4 to get a 20 percent tip). So 2.50 x 3 = 7.50. There. Much easier than multiplying something by 15 percent.

The problem many people have with algebra is simply putting what they do all the time into mathematical notation. So don't think in terms of notation! It's still algebra.

Logarithms (What the...?)

Upon seeing the word "logarithm," many people think, "That can't be good." First, let's see what it is:

$$\log_b x = y$$

where

$$x = b^y$$

Wow. That can't be good.

The above shows that a logarithm is the inverse of exponentiation. Exponentiation is the idea of multiplying a number by itself some number of times; a logarithm gives us the number of times we did that multiplication. It's the way banks calculate interest and the way scientists analyze growth (as we'll see in an example shortly).

Before we all panic, though, let's put in some real numbers. Make $x = 100$ and $b = 10$. This means that y must be 2, because $100 = 10^2$. That's exponentiation, which isn't so bad. So what's a logarithm? It's the opposite: $\log_{10} 100 = 2$. In other words, if 10 is the base b, what does the exponent (the power, y) have to be in order to get 100? Asked another way, how many times did we multiply 10 by itself to get 100?

The answer is 2.

We can think of exponents this way:

$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
etc.

Similarly, we can think of logarithms this way:

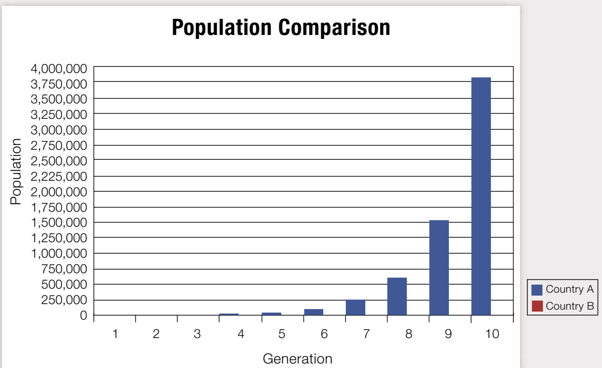
$\log_{10} 10 = 1$
$\log_{10} 100 = 2$
$\log_{10} 1000 = 3$
etc.

In essence, when $b = 10$, you can find the logarithm just by counting the zeros in the number. Of course, this only works when the number starts with a 1 followed only by zeros. (For example, $\log_{10} 250 = 2.39794$, but what fun is that?) But how is this useful?

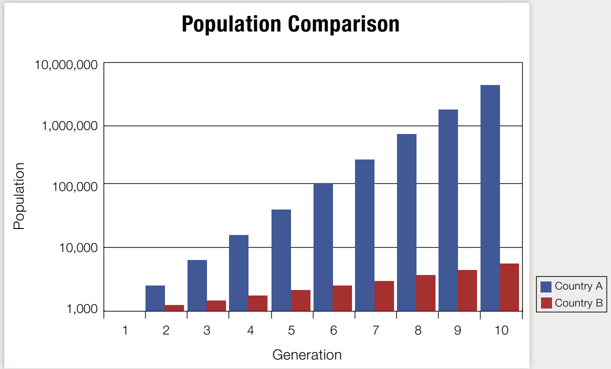
Suppose you want to compare the populations of two countries, A and B, and you know that their growth rates were 2.5 and 1.2, respectively. If both countries start with a population of 1000, the following chart shows their populations after 10 generations:

Generation	Country A	Country B
1	1000	1000
2	2500	1200
3	6250	1440
4	15625	1728
5	39063	2074
6	97656	2488
7	244141	2986
8	610352	3583
9	1525879	4300
10	3814697	5160

That's fine, but it is often more instructive to show things graphically. Creating a graph in Excel produces:



Country A's population grows at a much higher rate than B's, which leads to A's high population at generation 10. However, B's information is completely lost when we chart it this way. Now that we know something about logarithms, the y-axis labels can be changed to a logarithmic scale in Excel. This means that instead of adding an increment from one y value to another (the increment is 250,000 above), the previous y value is multiplied by ten:



Now the scale grows right along with the population, and thus all of the data can be viewed and compared. Nice.

Another example of the use of logarithms, and also a timely one in the wake of the recent tsunami disasters, is the Richter Scale, used to measure the magnitude of an earthquake. The Richter Scale goes from 1 to 9, but is logarithmic. Each increase of one point in magnitude means a tenfold increase in the size of the seismic waves. For example, a magnitude 5.0 earthquake is ten times "bigger" than one with a magnitude 4.0 on the Richter Scale.

Statistics

Statistics are ubiquitous. They are sprinkled throughout television newscasts, newspaper articles, and scholarly articles. For example, on page D1 of *The Boston Globe* on February 26, an article stated, "The median sale price for single-family homes was $340,450 in January." Most people understand the notion of *mean*, or average, but it's usually the term *median* that is used in this context. Here is a good example where the statistics are quite easy and useful, but the notation is daunting.

Suppose there are five houses, priced as follows:

House	Price
1	300,000
2	340,000
3	345,000
4	350,000
5	360,000

To find the average, we add all of the house prices together and divide by five. In mathematical notation, this is:

$$\bar{x} = \frac{\sum_{i=1}^{5} x_i}{5}$$

Good gracious! It seems daunting, but this is just notation for something you already know how to do! The $\sum$ means "add up"; the $i = 1$ means start at the first item, and the 5 at the top means end at the fifth item. The x represents a house price. Because we have five different prices, the first house is represented as x_1, which means that the i is replaced by 1. The second house is represented as x_2, and so forth. After all of the prices are added together, the sum is divided by five to get the mean, represented by $\bar{x}$.

In any case, the mean price of the list of houses is $339,000. Now let's compare that to the median, or M. The median is the *middle* value of an ordered list of items. Since we have five items, House 3 is the middle. If there are an even number of items in a list, the median is found by averaging the two middle values.

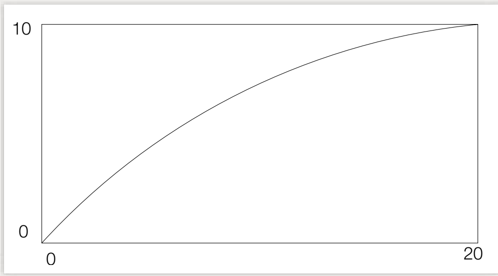
So, the median is simply $345,000. Note that the mean and the median are fairly close, in this case. Why did the *Globe* use median instead of mean? Because it's a better indication of the middle. Why? Suppose that the price of House 5 is $700,000 instead of $360,000. The mean house price now jumps to $407,000. However, the median *stays the same* at $345,000. For the average person who's wondering about how much a house is likely to cost, this median value is much closer to most of the houses than the mean, which is inflated because of the one high-priced house.

Calculus (You've gotta be kidding...)

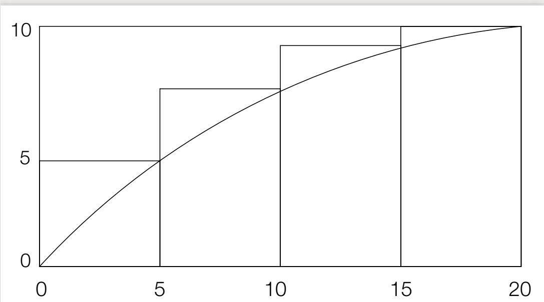
Calculus is often thought of as being something mystical, magical or horrific, depending on one's point of view. However, calculus is built upon two rather simple ideas that are anything but mystical, magical or horrific. That's not to say that it's all easy, but at least the basic premise is not a big deal. The two problems are:

- Finding the slope at any point on a given curve. This is the kind of problem that an engineer might be interested in, such as determining how steep a road will be at any point along a hill.
- Finding the area under a curve. Let's look at this one in more detail.

Suppose you wanted to paint a 20 x 10 wall in your daughter's room. You want to buy only as much paint as is necessary, so you figure out the area to paint, which is the length x the width, or 200 square feet. But, your daughter comes in and says she wants to paint the new color only below this curve:



Now the problem is more difficult, because the area to paint is no longer a simple rectangle. Knowing how to calculate the area of a rectangle still helps, however. Let's split the wall into four rectangles, where the upper right corner meets the paint demarcation line:

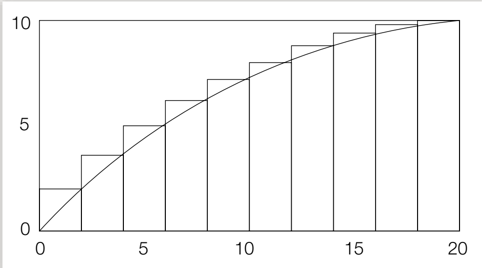


We can calculate an estimate of the area by adding together the areas of the four rectangles (*y* values are approximate):

area = 5 x 5 + 5 x 7 + 5 x 9 + 5 x 10 = 155 square feet

Although 155 square feet is an estimate, it is much closer to the true area than the original measure of 200 square feet for the entire wall, and probably closer than if you just guessed. However, note that all of the rectangles are a bit above the line, so that the estimate is a little on the high side. This estimate is sometimes called the *upper sum*. Instead of making the upper right corner of each rectangle touch the line, you could make the upper left corner touch the line. This would make each rectangle slightly too small, and adding up those sums would give you the *lower sum*.

In order to get a better estimate, we simply make additional, smaller rectangles:



Adding up all these rectangles gives us approximately 142 square feet. This is a little less than our first estimate, which makes sense because the rectangles do not protrude above the line as much as they did previously. As the number of rectangles increases, the computed area becomes more and more precise. Using calculus techniques, the area under the curve above turns out to be just over 134 square feet. (For you really mathy types, the function I used was $f(x) = 2.25\sqrt{x}$.) This is the basic idea behind much of calculus! Representing the left end of the wall as *a* and the right end as *b*, the way to represent this problem is:

$$area = \int_a^b f(x)\,dx$$

You already understand how to solve the problem, so don't let the notation make you think that suddenly things have gotten crazy. The $\int_a^b$ means find the area between *a* and *b*. The *f(x)* represents the *y* where the rectangle touches the line; it's the same notation we saw back when we looked at algebra. The *dx* refers to the fact that we are using a value of *x* in order to find *y* (as opposed to using *y* in order to find *x*).

So, there you have it. You understand a little bit of calculus. Not all that mysterious, is it?

It can be difficult teaching mathematical concepts to students who sometimes con themselves into believing they aren't "math people." This difficulty is offset, though, when a student "gets it." It's gratifying to see the satisfaction that occurs when students in a statistics course understand what the margin of error in a poll is all about. Or when students in a computer science course can apply an idea like a logarithm to a problem, as we did above. Or how calculus—yes, even calculus—has applications in economics problems. We can, and should, expect this kind of understanding from our students and ourselves. Math is everywhere; we just have to remain open to learning.